Nonlinear Analysis of Composite Plates and Shells subjected to in-plane loading

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***Abstract*—in this article, nonlinear analysis of the composite plates were carried out using the semiloof shell element. The finite element formulation is based on Green strains and Piola-Kirchhoff stresses. The nonlinear solution procedure was implemented to study the nonlinear behaviour of composite plates. Due to coupling effect in composite plate and shells under in-plane load, pre-buckling displacement is significant and hence the behaviour is nonlinear. A verification study has been carried out to establish the efficiency of the present model. Since the margin of factor of safety is less in aerospace application, the detailed understanding and study of pre-buckling displacement is necessary for the designer**

***Keywords*—**Semiloof; Nonlinear; Finite element; composite; plate.

I. INTRODUCTION

Composite materials possess very high specific properties and excellent fatigue and impact strengths. In unsymmetrical laminated composite plates as soon as the in-plane load is applied, it undergoes lateral displacement even though the load applied is much lower than the buckling load which was pointed out by Leissa [1]. This is termed as pre-buckling deformation.

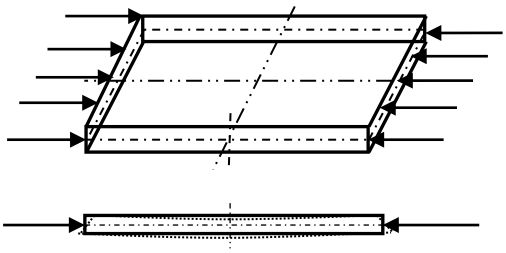

Fig.1 Pre-Buckling displacement in unsymmetrical laminated plates due to in-plane loads

The magnitude of the pre-buckling deformation depends on bending-stretching and types of coupling in the laminate. The extent of coupling depends on the number of layers in the laminate. The buckling analysis of a laminate with the prebuckling deformation included becomes analogous to that of plate with geometric imperfection, which is a nonlinear problem. If the prebuckling deformation are ignored it becomes a linear buckling problem. To investigate this, the Composite plates subjected to in-plane loads are studied here treating them as:(i) linear buckling (ii) extended linear buckling and nonlinear buckling.

Cetkovic et al. [2] performed a nonlinear analysis of laminated plates using layer wise displacement model, in which the nonlinear incremental algebraic equilibrium equations are solved using direct iteration method. Wang et al. [3] analysed the displacement and stress analysis of laminated composite plates using eight node quasi-conforming solid shell element, which is not only locking free but is highly computational efficiency as it possesses the explicit element stiffness matrix, where all the six components of stresses can be evaluated by the element in terms of 3-D constitutive equations and appropriately assumed element strain Faria [4] investigated the buckling optimization and pre-buckling effect of a composite plate using piezoelectric actuators, where the actuators are used to achieve two goals: to optimize buckling load under uncertain loading via stress stiffening effects and to ameliorate the plate prebuckling response through application of piezoelectric bending moments. Han et al. [5] deals with post-buckling behaviour of laminated composite plates under the combination of in-plane shear, compression and lateral loading using element-based Lagrangian formulation where the natural coordinate based strains, stresses and constitutive equations are used in the element and it uses only single mapping but it also converges faster. Vanet al. [6] improved the finite element computational model using a flat four-node element with smoothed strains for geometrically non linear analysis of composite plates. The vonkarman's large deflection theory and total Lagrangian approach is employed to describe the small strain geometric nonlinearity with large deformation using (FSDT).Cheon[7] used Lagrangian formulation for nonlinear analysis of shell structures where the stress-strain and constitutive equations are based on the natural coordinate has been used throughout the formulation which offers advantages of easy implementation compared to the traditional Lagrangian formulation.

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II. FINITE ELEMENT FORMULATION

The Finite Element Formulation is based on principle of virtual work.

Internal Work done by stresses =External forces due to virtual Displacement.

$$\int \delta \bar{e}^T \sigma d\, v = \int \delta u^T . p.\, da \qquad (1)$$

σ = Stress Vector ,$\bar{e}$ = Strain vector .
u =Displacement component vector,
p=Externally Applied load,
da=Elemental area ,dv=Elemental volume
Linear stress strain relation is expressed as

$$\sigma = [Q](\bar{e} - e_T) \qquad (2)$$

$[Q]$ = Transformed Reduced stiffness Matrix ,
e_T= Initial strain due to temperature Rise
$e_T = \alpha T$ *(3)*
α-Coefficient of thermal Expansion,
T-Rise in Temperature

The strain at any point in FGM plates is written as [5]

$$\bar{e}_{xx} = e_{xx} + zK_{xx} \qquad (4)$$
$$\bar{e}_{yy} = e_{yy} + zK_{yy} \qquad (5)$$
$$\bar{e}_{xy} = e_{xy} + zK_{xy} \qquad (6)$$

Where

$$e_{xx} = U_x + \frac{1}{2}\left[U_x{}^2 + V_x{}^2 + W_x{}^2\right]$$
$$e_{yy} = U_y + \frac{1}{2}\left[U_y{}^2 + V_y{}^2 + W_y{}^2\right]$$
$$e_{xy} = U_y + V_x + \left[U_x U_y + V_x V_y + W_x W_y\right]$$
$$K_{xx} = W_{xx} + \frac{1}{2}\left[W_{xx}{}^2 + W_{xy}{}^2\right]$$
$$K_{yy} = W_{xx} + \frac{1}{2}\left[W_{yy}{}^2 + W_{xy}{}^2\right]$$
$$K_{xy} = 2W_{xy} + \frac{1}{2}\left[W_{xy}(W_{xx} + W_{yy})\right]$$

Where U_x denotes derivative of U w.r.to x,
We can Write

$$[e] = \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{bmatrix} and [k] = \begin{bmatrix} k_{xx} \\ k_{yy} \\ k_{xy} \end{bmatrix}$$

The left-hand side of Eq.(1) may be written as

$$\int \delta e^{-T} \sigma d\, v = \int ([e] + z[k])^T\, [\bar{Q}](\bar{e} - e_T) dv \qquad (6)$$

$$= \int ([e] + z[k])^T\, [\bar{Q}]([e] + z[k] - e_T) dv$$
$$[\because = \bar{e}([e] + z[k])]$$
$$= \int ([e] + z[k])^T\, [\bar{Q}]([e] + z[k] dv$$
$$-([e] + z[k])^T [\bar{Q}] \alpha T dv \qquad (7)$$

For the composite volume integral is split in to two parts,
integrating [3,4]

$$\int \delta \begin{bmatrix} e \\ k \end{bmatrix}^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} e \\ k \end{bmatrix} da - \int \delta \begin{bmatrix} e \\ k \end{bmatrix}^T \begin{vmatrix} F_N \\ M_T \end{vmatrix} da \qquad (8)$$

The [A][B] and [D] matrices are called as the extensional stiffness, coupling stiffness, bending stiffness respectively.

$$([A],[B],[D]) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, z^2)[Q]\, dz \qquad (9)$$

And the thermal force F_N and
the thermal moment M_T are given by

$$\{F_N, M_T\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} [Q]\{\alpha(z)\} \Delta T(1, z) dz \qquad (10)$$

We can write

$$\begin{bmatrix} e \\ k \end{bmatrix} = [e_L] + [e_{NL}] \qquad (11)$$

e= plain strain, k =curvature,
$[e_L]$ = Linear part ,
$[e_{NL}]$=Non Linear part
The linear vector is

$$[e_L] = \left[u_x, v_x, (u_y + v_y), w_{xx,} w_{yy,} 2w_{xy}\right]$$

The nonlinear part can be written as

$$[e_{NL}] = \frac{1}{2}[R_o][\phi] \qquad (12)$$

Where φ is the vector of slope and defined as

$$[\phi]^T = [u_x, u_y, v_{x,} v_y . w_x . w_{y,} w_{xx,} w_{yy}, w_{xy}\ ]$$

Where

$$[R_o] = \begin{bmatrix} u_x & 0 & v_x & 0 & w_x & 0 & 0 & 0 & 0 \\ 0 & u_y & 0 & v_y & 0 & w_y & 0 & 0 & 0 \\ u_y & u_x & v_y & v_x & w_y & w_x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & w_{xx} & 0 & w_{xy} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_{yy} & w_{xy} \\ 0 & 0 & 0 & 0 & 0 & 0 & w_{xy} & w_{xy} & 0 \end{bmatrix} \qquad (13)$$

By taking the variation in Eq. (11)

$$\delta \begin{bmatrix} \varepsilon \\ k \end{bmatrix} = \delta[\varepsilon_L] + \frac{1}{2}[R_o]\delta[\phi] + \frac{1}{2}\delta[R_o][\phi]$$
$$= \delta[\varepsilon_L] + [R_o]\delta[\phi] \qquad (14)$$

Where

$$[R_o]\delta[\phi] = \delta[R_o][\phi]$$

The vector displacement from shape function matrix of Semiloof shell element is [3,4]

[u] = [d] [q] *(15)*

Where [q] - Nodal degree of freedom.
[d] - Shape function

The vector of slope$[\phi]$can be written as

$$[\phi] = [G][q] \qquad (16)$$
$$\delta[\phi] = [G]\delta[q] \qquad (17)$$

The strain energy displacement relation for linear part is given as

$$[e_L] = [B_L][q] \qquad (18)$$
$$[e_L] = [B_L]\delta[q]$$

Using Eq. (13) and Eq.(17)

$$[e_{NL}] = [R_o][G]\delta[q] \qquad (19)$$

Therefore the nonlinear strain matrix$[B_{NL}]$ can be written as

$$[B_{NL}] = [R_o][G] \qquad (20)$$

Substituting Eq.(17), Eq.(18) and Eq.(19) in Eq. (14)

$\delta \begin{bmatrix} \varepsilon \\ k \end{bmatrix} = [[B_N] + [B_{NL}]]\delta[q]$

$= [H]\delta[q]$ *(21)*

$H = [B_N] + [B_{NL}]$ *(22)*

The finite element representation of FGM plates using the Equations

$$\int \delta \bar{e}^T \sigma dv = \int \delta u^T P da$$

$\sigma = [Q](\bar{e} - e_T)$ with some simplifications can be written as arbitrary variations in [q] for single element. $\int \delta[q]^T [H]^T [F_0] da = \int [q]^T \lfloor \int [q]^T [P] \rfloor da + \int [H]^T [F_T] da$ *(23)*

Where $[F_0] = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} e_L \\ e_{NL} \end{bmatrix}$,

$[F_T] = \begin{bmatrix} F_N \\ M_T \end{bmatrix}$ and $[F] = [F_o] - [F_T]$ *(24)*

Since $\delta[q]$ is an arbitrary variation of nodal displacement, the non linear equation for composite plates and shells reduced to

$\psi = \int [H]^T [F_0] da - [F_o] - [F_T] = 0$ *(25)*

where ψ is the vector residual force.

$[f_m] = \int [d]^T [p]\, da$ *(26)*

$[F_T] = \int [H]^T \begin{bmatrix} F_N \\ M_T \end{bmatrix} da$ *(27)*

Assuming the solution in the current configuration known as $\bar{q}$,[6]the approximation of *Ψabout q is*

$\psi(\bar{q}) = \psi(q + \triangle q) = \psi(\bar{q}) + \left[\frac{\delta\psi}{\delta q}\right]_q \delta q + \cdots = 0$

(28)

Ignoring the higher order terms, a first order approximation relating the vector of residual forces to the displacement increments is obtained at 'q'= $(q + \triangle q) = [K_T] \triangle q$ *(29)*

where $[K_T]$ is the tangent stiffness matrix.

$[K_T] = \left[\frac{\delta\psi}{\delta q}\right]_{q=\bar{q}}$ *(30)*

Solution of linear Eq. (29) provides vector of displacement increments and therefore the solution is in the future configuration, Since the linear equation is only a first order approximation to the original nonlinear Eq. (13), iteration must be carried out with an increment to obtain more accurate results. Assuming the solution obtained at the i^{th} iteration is q(i) then the new approximation solution is

$\bar{q}(i+1) = \bar{q}(i) + \triangle q(i)$

(31)

The solution is exact $\bar{q}(i+1)$I exact if $\bar{q}(i+1) = 0$

The explicit expression for tangent stiffness $[K_T]$ in terms of previously determined element matrices can be determined from Eq.(13)

$$\delta\psi = \int [H]^T \delta[F_0] da + \int \delta[H]^T [F_0] da - \int \delta[H]^T [F_T] da$$

$= \int [H]^T \delta[F_0] da + \int \delta[H]^T [F] da$ *(32)*

From Eq.(11) and Eq.(22)

$\delta[H] = \delta[B_{NL}] = \delta[R_o][G]$ *(33)*

Substituting Eq. (17) and Eq. (33) in Eq. (32)

$\delta\psi = \int [H]^T [E][H] da \delta q + \int [G]^T \delta [R_0]^T F da$

(34)

Where $[E] = \begin{bmatrix} A & B \\ B & D \end{bmatrix}$

Expanding $[R_0]^T [F] = P\delta[\phi] = [P][G]\delta[q]$ *(35)*

$$\textit{Where} \quad [P] = \begin{bmatrix} N_{XX} & N_{XY} & 0 & 0 & 0 & 0 \\ N_{XY} & N_{YY} & 0 & 0 & 0 & 0 \\ 0 & 0 & N_{XX} & N_{XY} & 0 & 0 \\ 0 & 0 & N_{XY} & N_{YY} & 0 & 0 \\ 0 & 0 & 0 & 0 & N_{XX} & N_{XY} \\ 0 & 0 & 0 & 0 & N_{XY} & N_{YY} \end{bmatrix}$$

(36)

Substituting in Eq. *(27)*

$$\delta\psi = \int [H]^T [E][H] da \delta[q] + \int [G]^T [P][G] da \delta[q]$$

$= [K_T]\delta[q]$ *(37)*

Substituting for *[H]*=$[B_L] + [B_{NL}]$,*From Eq.(14)*

$$\int \left[[B_L] + [B_{NL}]\right]^T [E] \left[[B_L]\right] + [B_{NL}]] da \delta[q] + \int [G]^T [P][G] da \delta[q]$$

$$= \int [B_L]^T [E][B_L] da\delta[q] + \int [B_L]^T [E][B_{NL}] da\delta[q] + \int [B_{NL}]^T [E][B_L] da\delta[q] \int [B_{NL}]^T [E][B_{NL}] da\delta[q] + \int [G]^T [P][G] da\delta[q]$$ *(38)*

$= [K_T]\delta[q]$

The Tangent stiffness matrix is given by,

$[K_T] = [K_L] + [K_{NL}] + [K_G]$ *(39)*

Where $[K_L]$ is a linear structural stiffness matrix.

$[K_L] = \int [B_L]^T [E][B_L] da$ *(40)*

$[K_{NL}]$ is initial displacement matrix or Large displacement matrix or nonlinear stiffness matrix.

$$[K_{NL}] = \int [B_L]^T [E][B_{NL}] da + \int [B_{NL}]^T [E][B_L] da + \int [B_{NL}]^T [E][B_{NL}] da$$

(41)

$[K_G] = \int [G]^T [P][G] da$ *(42)*

$[K_G]$ is geometric stiffness matrix or initial stress matrix.

The most common approximation of the nonlinear problem in buckling analysis treating the prebuckling behaviour as linear and taking $[K_{NL}] = 0$

$$\therefore \delta\psi = [K_L] + [K_G]\delta[q]$$

If the loads are increased by a factor λ we find that a neutral stability exists [3]. That is

$\left[[K_L] + \lambda[K_G]\right]\delta[q] = 0$ *(43)*

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From this λ can be obtained by solving the typical Eigen value problem

$|[K_L] + \lambda[K_G]| = 0$ (44)

In Fig .1 this corresponds to bifurcation on point "a"

The next improvement considers the initial displacements matrix $[K_{NL}]$ as linear (that is, prebuckling deformation is linear)

Which leads to the extended Eigen value problem.

$|[K_L] + [K_{NL}] + \lambda[K_G]| = 0$ (45)

In Fig. 1 the buckling load corresponds this to point " b".If the prebuckling deformation is nonlinear, it will become nonlinear analysis and the buckling load corresponds to point "c" or "d. If it is a case of bifurcation buckling the load corresponds to point 'c '. If it is a limit load case, then buckling load corresponds to point 'd'.

CONVERGENCE AND VALIDATION: The program developed using Semiloof shell element by Singh and Thangaratnam[10,11] for thermal stress, vibration and buckling analysis of isotropic, composite plates and shells is extended to nonlinear analysis based on the above formulation. The program is validated with results available in the literature and good agreement is observed.The boundary conditions given in Ref.[10,11]are used.

SS1Simply supported u=0,v=0,w=0, $\theta xz \neq 0$, at x=0,a and u=0,v=0,w=0, $\theta xz \neq 0$,at y=0,b

SS2 Simply supported u≠0,v =0,w=0, $\theta xz \neq 0$, at x=0,a and u =0,v ≠0 ,w=0, $\theta xz \neq 0$,at y=0,b

III. RESULT AND DISCUSSION

Square plate of size 100mmx100mm and thickness h =1 mm. (a/h=100) is considered. The linear and non-linear buckling analysis of the plate is been studied here using different boundary conditions such as SS1 and SS2, the Material property: E_{II}/E_{TT} = 40, G_{IT}/E_{TT} = 0.5, μ_{IT} = 0.25, E_{TT} = 1.0 x 10^4 Kg/cm^2 and a/h = 100 is considered and analysed which consist of number of layers from 2 to 6 layers.

Mechanical Load: Squareunsymmetrical cross-ply laminates with three type of boundary condition (S1and S2) has been studied. The buckling behaviour of the square laminate has been studied using the present formulation treating it as linear buckling where in the prebuckling deformation is ignored and also as extended linear buckling where in the prebuckling deformation is treated as linear function of load.

Table 1 Linear Buckling loads for simply supported (S1) square plates

LINEAR BUCKLING LOADS FOR SIMPLY SUPPORTED (S1) SQUARE PLATES	
NO.OF LAYERS	LINEAR BUCKLING LOAD
2	22.9
4	35.9
6	37.6

The above table describes the difference in the level of Linear buckling (SS1) based on the number of layers.

Simply supported (SS2) boundary condition due to in-plane displacement unrestrained: The central displacement vs. in plane load with the number of layers as parameter is presented. The square plate is considered as both symmetric and unsymmetrical cross-ply and angle-ply, using SS2 boundary condition.

The Linear buckling analysis of cross-ply square plate has been studied and is mentioned in the below table,

Table 2 Linear Buckling loads for simply supported (S2) square plates-cross ply

LINEAR AND NON-LINEAR BUCKLING LOADS FOR SIMPLY SUPPORTED (S2) SQUARE PLATES	
NO.OF LAYERS	LINEAR BUCKLING LOAD
2	12.42
4	27.81
6	26.9

Square plate is also analysed in the form of angle ply for symmetric and unsymmetrical condition using SS2 boundary condition. The analysis is done by considering different kinds of angles staring from 15° to 90°. The below table describes the linear analysis of the plate, and the graph is plotted for number of layers vs. buckling values of different layers.

Symmetric condition: square plate (angle ply) is considered and the linear buckling analysis results are taken.

Table 3.Linear buckling analysis -Laminated (angle ply-symmetrical SS2)

Laminated-Angle Ply (symmetrical-S2)						
No.of layers	15°	30°	45°	60°	75°	90°
2	116	1772	2305	1778	1907	6442
4	278	574	911	1253	4332	6442
6	615	656	1066	1365	5113	5690
8	757.	855	1457	1675	4722	6442

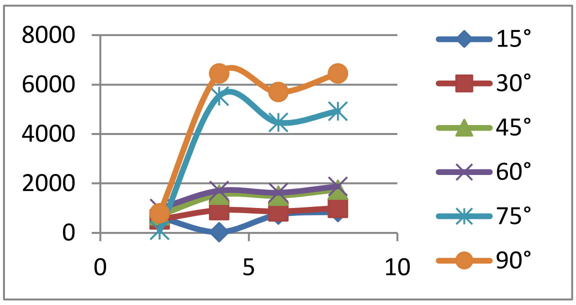

Fig 2. Symmetric laminated angle ply-linear buckling

The linear buckling analysis of symmetrical laminated angle-ply has been studied for number of layers (2 to 8) with different angle from 15° to 90°. From the above results its noted that (2-Layer) has the maximum buckling effect on the angle ply-square plate.

unsymmetrical condition : square plate (angle ply) is considered and the linear buckling analysis results are taken.

The buckling (Linear) analysis of unsymmetrical condition has been studied , the results are taken for number of layers (2 to 8) consisting of angle variation from 15° to 90°. Table 4. describes the results derived from SS2-boundary condition.

Table 4.Linear analysis of laminated angle ply-(Unsymmetrical-S2)

Laminated-Angle Ply (unsymmetrica-S2)						
No.of layers	15°	30°	45°	60°	75°	90°
2	628	544	710	984	102	793
4	278	917	1539	1704	5532	6442
6	736	868	1499	1622	4462	5690
8	843	1003	1749	1879	4924	6442

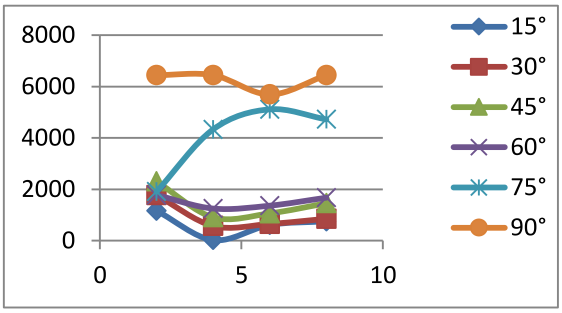

Fig 3. Unsymmetrical laminated angle ply-linear buckling

Non-linear Buckling analysis: The central displacement vs. in plane load with the number of layers as parameter is presented. The nonlinear buckling analysis results for cross-ply square plate is studied and compared with the linear buckling analysis.

Table 5. Nonlinear Buckling analysis of laminated cross-ply

LINEAR AND NON-LINEAR BUCKLING LOADS FOR SIMPLY SUPPORTED (S2) SQUARE PLATES		
NO.OF LAYERS	LINEAR BUCKLING LOAD	NON-LINEAR BUCKLING LOAD
2	12.42	43.02
4	27.81	30.68
6	26.9	25.84

From the above results it is noted that the linear analysis at different layers has higher variation of buckling , in terms of nonlinear buckling analysis the variation is higher thanof the linear buckling at different layers.

Thermal load: 2-layer cross-ply square laminate subjected to uniform temperature rise under types of boundary condition (S1,S3,C1,C3) have been analysed using linear and nonlinear formulations. The results are presented below,

Table 6. Critical Temperature T_{cr} for (2-layers) cross-ply laminated square plates.

LINEAR AND NON-LINEAR BUCKLING ANALYSIS FOR CRITICAL TEMPERATURE		
NO.OF LAYERS	LINEAR BUCKLING LOAD	NON-LINEAR BUCKLING LOAD
S1	82.7219	82.7219
S3	82.8125	82.8125
C1	109.7077	109.7077
C3	109.6990	109.0400

The linear and nonlinear analyses give rise to almost same buckling loads. there is a difference between the values under simply supported and clamped boundary conditions with the latter having the higher value.

IV.CONCLUSION

It is observed here that the nonlinear formulation is required if the prebuckling deformation is considerable and linear bifurcation maybe expected to give satisfactory results if the prebuckling deformation are small. The extent of prebuckling deformation depends on parameters such as number of layers, boundary conditions and type of load. In the case of laminates with large number of layer >8, the linear analysis is sufficient irrespective of other parameters. A static stress analysis may be expected to reveal the fact whether linear or nonlinear buckling analysis is required in a specific case. The behaviour of laminated composite plates under thermal load are different from that under mechanical loads whether unidirectional or bidirectional. this is due to the dependence of the thermally induced loads on parameters like fibre orientation angle and stacking sequence.

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